

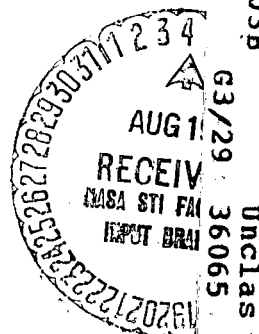
SOUNDING OF THE SOLAR CORONA BY MONOCHROMATIC RADIO RADIATION

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## SOUNDING OF THE SOLAR CORONA BY MONOCHROMATIC RADIO RADIATION

O. G. Onishchenko

ABSTRACT. A method of sounding the solar corona by monochromatic radio radiation on two coherent frequencies is reviewed. Estimates of regular group delay, of fluctuation in group delay, of rotation of the plane of polarization, and of expansion of the signal spectrum as a function of the beam's impact parameter for the accepted model of the solar corona and the interplanetary medium, are cited. The inverse problem of obtaining certain mean distributions of electron concentrations, magnetic field, and solar wind velocity as a function of the heliocentric distance is reviewed.

1. Introduction

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Flights of space stations into interplanetary space make it possible to measure the parameters of the solar plasma by direct methods. The region accessible for direct measurements is contained between the orbits of Venus and Mars ( $0.7 \text{ AU} < r < 1.5 \text{ AU}$ ) and lies near the plane of the ecliptic (see Figure 1). The substantial fluctuations in the parameters of the interplanetary plasma (concentration  $N$ , solar wind velocity  $V$ , and others) in the region of direct measurements do not permit making an adequately reliable determination of the distribution of certain of the means of these parameters as a function of the heliocentric distance,  $r$ . Theoretical papers dealing with the solar corona and the solar wind usually operate with certain mean distributions of the parameters  $N(r)$ ,  $V(r)$ , and temperature,  $T(r)$  [1, 2]. A very interesting experiment would be one that would result in obtaining simultaneously certain of the mean distributions of  $N(r)$  and  $V(r)$ , and, as a result, a certain steady plasma flux from the sun as a function of the heliocentric distance,  $r$ . This experiment would make it possible to reduce the number of possible theoretical models of the solar corona and solar wind, and thus take a step forward along the path to an understanding of these phenomena. Hence the results of a survey at radio wavelengths of the regions of the solar corona that are inaccessible to direct observations are highly interesting.

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\* Numbers in the margin indicate pagination in the foreign text.

There are three methods of observing the radio sources of the sun's eclipsing coronas.

The oldest method is that of observing the scattering of radiation from discrete natural radio sources. V. V. Vitkevich [3] and Machin and Smith [4], in 1951, made that assumption that it would be possible to determine the distribution of the electron concentration,  $N(r)$ , temperature,  $T(r)$ , and other parameters of the coronal plasma by observing radio radiation from the Crab Nebula when eclipsed by the solar corona, and to measure refraction, absorption, and other characteristics of the radio radiation. However, the interferometer observations yielded only obvious attenuation of intensity. It developed that intensity attenuation occurs as a result of scattering of radio waves by the inhomogeneities in the electron concentrations in the corona and far-out corona ( $r < 100 R_{\odot}$ ). The magnitude measured in the scatter experiments is the angle of scattering. The most complete theoretical research on the scattering of radio radiation in the solar corona has been done by Hollweg [5, 6, 7].

The discovery, in 1963, of quasars (sources with small angles  $\lesssim 1''$ ) made it possible to use a new method of radio scintillation to investigate the more distant regions of the corona [of the interplanetary plasma (see Figure 1)]. The idea behind this method is that radio waves passing through the layer of inhomogeneity of electron concentration create a diffraction pattern on earth. The movement of the inhomogeneity with respect to the earth causes movement of the diffraction pattern and creates a fluctuation in the intensity at the point of observation. Observation of scintillations at spaced points provide for determination of the size of the diffraction pattern and the rate of its displacement over the earth. These observations make it possible to judge the inhomogeneities in the electron concentration and the rates at which they move (the velocity of the solar wind, assuming freezing in). Hollweg and Jokipii [7, 8] have pointed out that the results of observations of scattering and scintillation do not contradict the spectrum of turbulence with a correlation scale of  $\sim 0.01$  AU and with an "internal scale" of turbulence of several hundred kilometers. The solar wind is very turbulent. These results concord well with direct observations of the solar wind, and differ significantly from previous estimates of inhomogeneities in the corona, and in the interplanetary plasma [9, 10, 5, 6]. Previous estimates assumed that the spectrum of the inhomogeneity in the corona was Gaussian with a correlation scale  $\sim$  several hundred kilometers, and with fluctuation in concentration in the

inhomogeneties  $\sim$  several percent relative to the mean value of  $N$ .

The third of the observation methods considers surveying the solar corona with monochromatic radio radiation from an artificial source [6, 11]. The source can be installed on board a space station flying around the sun, and the receiver can be installed on the ground. There can be the reverse as well, with signals measured on board the space station and the measurement results communicated to the ground. This method provides for obtaining information on regular (mean) electron concentration,  $N$ , with respect to group delay, as distinguished from the two preceding methods, which provide information on the fluctuation in the electron concentration,  $\Delta N$  (on the excess above the mean values in the inhomogeneities).<sup>1</sup> Moreover, measurement of the expansion in the signal spectrum,  $\Delta f$ , and in the rotation of the plane of polarization,  $\psi$ , for linearly /6 polarized radiation, makes it possible to judge the velocity of the solar wind and the magnetic field in the corona. The first results of the measurement of  $\Delta f$  and  $\psi$  now are available [13, 14].

The parameters of coronal plasma depend substantially on solar activity and on the solar latitude. This paper will review the feasibility of measuring solar plasma parameters  $N$ ,  $V$ , and  $H$  at the solar activity minimum, and near the plane of the solar equator.

## 2. Brief Survey of the Principal Results of Measurements of the Electron Concentration in the Solar Corona and in the Interplanetary Medium

### A. The regular electron concentration, $N_0$ , and the solar wind velocity, $V$

Figure 1 shows  $N(r)$ , obtained from optical observations and soundings. The electron concentration in the region of heliocentric distances out to  $\sim 20 R_\odot$  ( $R_\odot$  is the radius of the sun) is determined from optical observations of the solar corona. It is considered that the polarized component of the radiation from the corona in the continuous spectrum (the K component) can be attributed to the Thomson scattering of the radiation from the photosphere by the corona's free electrons. The difficulty in distinguishing the K component from the general radiation, and in determination of the electron concentration  $N(r)$ , increases with distance from the sun. The points in Figure 1 designate the values of the /7 electron concentration in the plane of the solar equator in the minimum activity

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<sup>1</sup> Pulsars too can be used for these observations, at least in principle. See [12].

case cited by K. de Yager ([15] in Table 39). The Allen-Baumbach empirical formula:

$$N_0(\rho) = 3 \cdot 10^{8-16} \rho + 1,5 \cdot 10^{8-6} \rho \text{ [electrons/cm}^3\text{]}, \quad (1)$$

in which  $\rho = r/R_\odot$ ,  $r$  is the heliocentric distance, often is used in estimates of electron concentration in the corona.

Flights of space stations in the region of the orbit of Venus, the Earth, and Mars have provided measurements of the ion (electron) concentration in this region. Direct measurements have established that fluctuations in electron concentration in this region are of the order of the mean magnitude [16]. The mean value of electron concentration in 1962 (solar activity minimum), gathered by Mariner 2, was  $\sim 5$  electrons/cm<sup>3</sup>. The mean value,  $N$ , increased to  $\sim 10$  electrons/cm<sup>3</sup> as the solar activity maximum approached. Measurements of integral electron concentration,  $\int_0^s N ds$  ( $s$  is the distance from the earth to the space station), from the measurement of the group and phase delays in the signal on two radio frequencies, were made in addition to the local  $N$  measurements [17, 18].

Solar wind velocity measurements yielded a mean of  $V \sim 350-400$  km/sec at minimum activity, with the relative fluctuations in the velocity in absolute terms less than the relative fluctuations in the electron concentration (relative to the mean value). The sounding measurements showed no significant change in  $V$  in terms of  $r$ .

Figure 1 shows two dashed lines, 1 and 2, with slopes of -2 and -2.5, the purpose of which is to connect the two regions in which the electron concentration was measured. The following were the considerations that led to the selection of the slopes of these lines. The equation for the conservation of mass flux, assuming the existence of a mean steady plasma flux and a spherically symmetrical flow, is:<sup>2</sup>

$$Nvr^2 = F_0 = \text{const.} \quad (2)$$

If, in addition, we add the assumption of constancy of the mean solar wind velocity in interplanetary space, what follows from Eq. (2) is

$$N(r) \sim r^{-2}.$$

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<sup>2</sup> One of the equations in the Parker system of hydrodynamic equations for solar wind [2].

And if  $N \sim r^{-2.5}$  is included, this relationship is a good description of the result of optical observations in the region of heliocentric distances,  $5 R_{\odot} \leq r \leq 20 R_{\odot}$ , and what follows from Eq. (2) is that  $v(r) \sim \sqrt{r}$ .

So there are two possible models of the distribution of the mean electron (ion) concentration (in the plane of the ecliptic at minimum solar activity) in the corona, and in interplanetary space, that can be used for numerical estimates:

$$N_{o1}(\rho) = 3 \cdot 10^8 \rho^{-16} + 15 \cdot 10^8 \rho^{-6} + 2.3 \cdot 10^5 \rho^{-2}, \quad (3)$$

$$N_{o2}(\rho) = 3 \cdot 10^8 \rho^{-16} + 15 \cdot 10^8 \rho^{-6} + 3 \cdot 10^6 \rho^{-2.5}, \quad (4)$$

where

$$\rho = r/R_{\odot}, \quad R_{\odot} \approx 7 \cdot 10^{10} \text{ cm.}$$

The terms  $\sim \rho^{-16}$  in Eqs. (3) and (4) can be ignored when  $\rho > 1.5$ .

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Figure 1 further shows the distribution of the solar wind velocity,  $V$ , obtained assuming satisfaction of Eq. (2) and electron concentration distributed in accordance with Eqs. (3) or (4).

Curve 3 in Figure 1 depicts the distribution of the electron concentration and the solar wind velocity in an isothermal corona for  $T = 10^6$  according to Parker [1].

## B. Solar Wind Turbulence

This paper has made the same assumption as to the spectrum of the electron concentration fluctuation density as that found in references [7, 8]. It has been assumed that the spectral density in plasma density fluctuations coincides with the measured spectral density of the magnetic field and with the velocity of the solar wind in earth orbit [7, 8]. It is not clear, a priori, that the spectrum of plasma density fluctuations has the same form as does the spectrum of fluctuations in the magnetic field, or in the velocity of the solar wind. But this should be anticipated if the magnetic field is frozen in the plasma. It should be pointed out that only the electron concentration has any influence on the propagation of electromagnetic waves. But because the characteristic scales of the inhomogeneities are larger than the Debye radius by many orders of magnitude, fluctuations in the electron concentration describe fluctuations in plasma density.

The turbulence is considered to be locally homogeneous and isotropic. If it is taken that the assumption as to the "freezing in" of the turbulence is feasible, a transition can be made from the measured one-dimensional time-dependent magnetic field spectrum,  $W_H(w)$ , to a spatial, three-dimensional, spectrum  $\Phi(q)$ , cm [19]: /10

$$\Phi(q) = -\frac{v^2}{2\pi q} W'(qv),$$

where  $v$  is the mean solar wind velocity, and  $q$  is the wave number.  $\bar{v} = 350$  km/sec for the calculations. Finally, the schematic three-dimensional spectrum of electron concentration fluctuations shown in Figure 6 is assumed.  $\alpha$  is an index for the degree of homogeneous spatial spectral density,  $V(q)$ , in the region  $q_1 < q < q_2$ , and ordinarily  $1 < \alpha < 2$ , [20]. Jokipii and Hollweg called the length corresponding to  $q_2$

$$L_2 = q_2^{-1} \approx 3.3 \cdot 10^2 \text{ km},$$

the "internal scale" for the turbulence. The correlation length

$$L_1 = q_1^{-1} \approx 1.7 \cdot 10^6 \text{ km},$$

corresponds to the wave number  $q_1$ . We use the relationship

$$\langle \delta^2 N \rangle = V(0) = \int_0^\infty V'(q) dq = \int_0^\infty 2\pi \Phi(q) q dq.$$

to obtain the connection between  $\Phi(q)$  and the root-mean-square fluctuation in electron concentration. When  $\alpha \geq 1$ , and  $q_2/q_1 = 10^4 \gg 1$ , and after integration, we obtain

$$\langle \delta^2 N \rangle = 4\pi \Phi_0 q_1^3 \left( \frac{1}{3} + \frac{1}{\alpha-1} \right).$$

### C. Large Scale Fluctuations in Electron Concentration and the Variability of the Integral Electron Concentration, $\mathcal{I}$ /11

The model of the distribution of the mean electron concentration in the solar corona, Eq. (3), or Eq. (4), with a spectrum of fluctuation density as described in the previous section, assumes a very homogeneous and stable injection of plasma from the solar surface. This contradicts the modern hypothesis with respect to the sun and the solar corona.

Local measurements of electron concentration [16] and measurements of the integral electron concentration, have established that there are fluctuations,  $N$ , with a period (with a characteristic time) of several days (sector structure).

These fluctuations have a more constant character; some repetition with the period of solar rotation of 27 days can be observed. These large-scale fluctuations can be regarded, approximately, as variations in the mean electron concentration with a period of several days.

Let us suppose that in the system of coordinates rotating with the sun the electron concentration,  $N$ , depends on  $r$  and  $\theta$  ( $\theta$  is the polar angle) as follows:

$$N(r, \theta) = f(\theta) \cdot N_0(r).$$

Angle  $\theta$  is connected with the angular rate of rotation of the sun,  $\Omega$ , by

$$\theta = \theta_0 + \Omega t.$$

From whence

$$N = f(t) \cdot N_0(r),$$

where  $f(t)$  is a periodic function. Thus, a rotating sector structure too leads to variability in  $N$  and  $\int N ds$  in a fixed system of coordinates. /12

Local measurements yield a greater time-dependent variation than do measurements of the integral electron concentration  $I = \int_0^S N dt$ . The integral concentration measured by the Pioneers along the track  $\leq 0.5$  AU changed from day to day within the limits of the coefficient 2, that is

$$I(t+1) = k I(t),$$

where  $I(t+1)$  is the integral electron concentration on the following day, and  $1/2 \lesssim k \lesssim 2$ . Indirect estimates of the change in  $I(t)$  can be made by using scintillation observations. Change in  $I$  by a factor of  $k$  corresponds to a change in the scale of the diffraction pattern by a factor of  $1/k$ , if the root-mean-square dephasing of successively radiating elements because of the inhomogeneity is  $\varphi_0 > 1$ . Radioastronomical data [21] lead us to expect a change in  $I$  from day to day with the coefficient  $k \leq 2-3$ , and usually less than 20 percent per hour.

There is also the variation in  $N$  associated with the 11-year solar cycle, and this is in addition to the variation in  $N$  with a period of a few days.  $N$  can increase by a factor of 2 to 3 with increase in solar activity as compared to the  $N$  value at minimum solar activity.

The preceding fluctuations describe the "normal" behavior of interplanetary plasma. Pioneer 7 observed large bunches of plasma leading to an increase in  $I$  by a factor of from 3 to 6 in about 4 hours, followed by a decrease to the normal

level [17]. Only three such sharp changes were observed between December 1965 and January 1967. The frequency of occurrence of these inhomogeneities increased from January 1967 to December 1967, with the approach of the solar activity maximum, and became an average of one a month.

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### 3. Determination Of Distribution Of Electron Concentration By The Method Of Surveying The Corona At Monochromatic Radio Wavelengths

#### A. The Method of Measuring the Integral Electron Concentration

$$\int_0^S N dl$$

Koehler, R. L. [17] has described in detail the procedure used to measure  $\int_0^S N dl$  in terms of the group delay, using radio frequencies to sound the interplanetary plasma from Pioneer 6 and Pioneer 7. Two powerful r-f signal carriers at frequencies  $f_1 = 49.8$  MHz and  $f_2 = 423.4$  MHz were transmitted from the ground to the spacecraft in deep space. These signals were continuously amplitude modulated by a frequency  $f_m = 8.692$  kHz (or 7.692 kHz). The group rate of propagation of electromagnetic waves in the plasma is connected with plasma frequency  $\omega_0$  and the wave's circular frequency,  $\omega = 2\pi f$ , disregarding the influence of the magnetic field ( $\omega \gg \omega_c$  is the hypofrequency of the electron) as follows

$$V_{gr} = c \cdot n = c \sqrt{1 - \omega_0^2 / \omega^2}.$$

Here  $n$  is the index of refraction,  $c$  is the speed of light. The time of the group delay between two signals with carriers  $f_1$  and  $f_2$  depends on the integral electron concentration

$$\Delta t_{gr} = \int_0^S \frac{dl}{V_{gr1}} - \int_0^S \frac{dl}{V_{gr2}} = \frac{10^8}{c} \cdot \frac{0.806}{2} \left( \frac{1}{f_1^2} - \frac{1}{f_2^2} \right) \int_0^S N dl.$$

in the high-frequency approximation ( $1 - n \ll 1$ ). Here  $c = 3 \cdot 10^{10}$  cm/sec,  $[\int_0^S N dl] =$  electrons/cm<sup>2</sup>,  $[\Delta t_{gr}] =$  sec. The time  $\Delta t_{gr}$  was measured in terms of the phase shift of the modulated amplitudes. The modulated shift with respect to time,  $t_m$ , is connected with the phase shift,  $\Phi$ , by

$$t_m = \frac{\Phi}{360} \cdot \frac{1}{f_m}$$

where  $\Phi$  is in degrees. The integral electron concentration  $\int_0^S N dl$  therefore is connected with  $\Phi$  as follows

$$I = \int_0^S N dl = \frac{\Phi}{360} \cdot \frac{1}{f_m} \cdot \frac{1}{(1/c) 0.403 [1/f_1^2 - 1/f_2^2]} = \quad (6)$$

$$= \Phi \cdot 0.6 \cdot 10^{14} \text{ electrons/cm}^2, f_m = 8.692 \text{ kHz}.$$

The phase modulated shift,  $\Phi$  was determined on board the spacecraft discretely with a spacing of approximately  $3^\circ$  and transmitted to the ground by telemetry. Thus, the integral electron concentration was determined discretely with a spacing (resolution) of approximately  $2 \cdot 10^{12}$  electrons/cm<sup>2</sup>. The ionosphere made a substantial contribution (50 to 80 percent) to the integral electron concentration in these experiments. The inclusion of data on the integral electron concentration for the ionosphere, obtained by radio sounding the ionosphere from satellites, made it possible to subtract the contribution of the ionosphere from the total integral concentration of electrons.

These  $\int N dl$  measurements were made in the interplanetary plasma in the region of heliocentric distances between 0.8 and 1.15 AU along the track S ~ 10 to 70 million km. The mean electron concentration in interplanetary space, from 125 measurements of the integral electron concentration made by Pioneer 6, was 4.3 electrons/cm<sup>3</sup>, with the root-mean-square fluctuation  $\pm 3.6$  electrons/cm<sup>3</sup> between December 1965 and May 1966. The 170 measurements of  $\int N dl$  by Pioneer 7 between August 1966 and March 1967, yielded a mean electron concentration of 8.7 electrons/cm<sup>3</sup>, and a root-mean-square fluctuation of  $\pm 4$  electrons/cm<sup>3</sup>. Greater inhomogeneity in plasma density was recorded in this period of greater solar activity. The quiet level of the mean electron concentration was approximately 5 electrons/cm<sup>3</sup>, corresponding to the measurements made of localized electron concentrations at this time.

#### B. Determination of the Mean Distribution of the Electron Concentration, $N_0(r)$ , in the Corona in Terms of Group Delay

Flights by space stations far out into space opened up new possibilities for studying the solar corona. The method of measuring the integral electron concentration in terms of group delay, used by the Pioneers, can be used to sound the denser regions of solar plasma. Figure 7 is a schematic presentation of the geometry of an eclipse. This paper at all times considers that the refraction of radio waves in the regular corona can be disregarded in the case of sufficiently short waves. Refraction of radio waves in the regular corona has been taken up in detail in reference [11]. Using Eq. (5), the expression for group delay, we can write

$$\Delta t_{gr} = A \int_0^s N_0 dl = 2A \int_R^R N_0(r) \frac{r \cdot dr}{\sqrt{r^2 - R^2}} \quad (7)$$

where  $A = \text{constant}$ . See Eq. (5) for given  $f_1$  and  $f_2$ .

Considering that  $N \sim r^{-2}$  in the interplanetary medium, the integral electron concentration takes the form /16

$$I_0(p) = 6,35 \cdot 10^{15} \frac{Q_2}{p} n_e \text{ (electrons/cm}^2\text{)}. \quad (8)$$

Here  $p=R/R_0$  is the impact parameter,  $n_e$  is the electron concentration in the earth orbit. See Figure 7.

We can put  $r_3 \rightarrow \infty$  in the integral in Eq. (7) when sounding denser regions of the corona, whereupon

$$\Delta t_{gr}(R) = 2A \int_R^\infty N_0(r) \frac{rdr}{r^2 - R^2}. \quad (9)$$

Substituting into Eq. 9 the models of the distribution of the electron concentration, Eqs. (3) and (4), and using

$$\int_0^\infty \frac{1}{p^n} \frac{p dp}{\sqrt{p^2 - p^2}} = \frac{\sqrt{\pi}}{2 p^{n-1}} \frac{\Gamma(\frac{n-1}{2})}{\Gamma(\frac{n}{2})} \quad (\Gamma \text{ is the gamma function}) \quad (10)$$

we obtain

$$\Delta t_{gr}(p) = A I_{1,2}(p), \quad / p = \frac{R}{R_0} /, \quad (11)$$

where

$$I_1(p) = 1,2 \cdot 10^{19} p^{-5} + 5 \cdot 10^{16} p^{-1} \text{ electrons/cm}^2, \quad (12)$$

$$I_2(p) = 1,2 \cdot 10^{19} p^{-5} + 5 \cdot 10^{17} p^{-1,5} \text{ electrons/cm}^2. \quad (12')$$

When  $f_1 = 100 \text{ MHz}$ , and  $f_2 = m f_1$ , where  $m \geq 3$ ,  $A$  is equal to

$$A \approx 1.3 \cdot 10^{-19} \text{ sec-cm}^2/\text{electron}.$$

Table 1 and Figure 2 list and show the results of the calculations made using Eqs. (12), (12'), and (11).

The maximum contribution of the ionosphere to the integral electron concentration is  $10^{14} \text{ electrons/cm}^2$  [22]. The contribution of the ionosphere to the measured total integral electron concentration must therefore be taken into consideration when sounding the outer corona. /17

Measurement of  $\int N_0(r) dr$  leads to a determination of some mean spherically symmetrical distribution in the electron concentration,  $N_0(r)$ , in the corona.

Eq. (9) can be solved for the inner regions of the corona. Eq. (9) is an integral equation of the Abel type, and its solution (inversion) is

$$N_0(r) = -\frac{1}{\pi A} \frac{d}{dr} \int_r^{\infty} \Delta t_{gr}(R) \frac{R}{R^2 - r^2} dR \quad (9')$$

or it can be demonstrated that

$$N_0(r) = -\frac{1}{\pi A} \int_r^{\infty} \frac{d}{dR} \Delta t_{gr}(R) \frac{dR}{R^2 - r^2} \quad (9'')$$

It should be pointed out that of equal interest is the trajectory of the space station (or of two stations) for which  $R = \text{constant}$ . Measurement of the group delay for this trajectory makes it possible to determine  $N$  in terms of the angle  $\Theta$  (and of time  $t$ ); that is, to isolate the long-lived sector structure of the corona.

### The Dispersion Interferometer Method

The dispersion interferometer method permits us to measure  $d/dt \int N dl = \dot{I}$  [11]. The integral electron concentration,  $I$ , is a function of the impact parameter,  $P$ , and the time,  $t$ , in the corona model considered; that is,  $I = I(p, t)$ . Therefore

$$\frac{d}{dt} I(p, t) = \frac{\partial}{\partial p} I \frac{dp}{dt} + \frac{\partial}{\partial t} I = G_1 + G_2,$$

where

$$G_1 = \frac{\partial}{\partial p} I \frac{dp}{dt}, \quad G_2 = \frac{\partial}{\partial t} I.$$

We can use the Pioneer 6 trajectory during its eclipse by the corona to estimate  $dp/dt$  [14]. The 24-hour change in  $\Delta p$  was  $\approx 0.5$  cm [11]. Using the results listed in Table 1, we can estimate  $G_1$ . The  $G_2$  estimates are based on the assumption that  $I$  can increase by a factor of 2 in 24 hours. The results are listed in Table 2. Comparing  $G_2$  and  $G_1$  in Table 2, we see that they can be of the same order of magnitude, or that  $G_2 > G_1$ . So,  $G_1$  cannot be isolated from the experimental measurements made in this solar corona, and the  $N_0(r)$  distribution cannot be obtained.

The maximum rate of change in the integral concentration in the ionosphere at minimum activity is a magnitude of the order of  $10^9$  electrons/cm<sup>2</sup>-sec.

The Doppler shift in the signal frequency is related to  $d/dt \int N dl$  as follows [11]

$$\Delta f = \frac{e^2}{mc} \cdot \frac{1}{f} \cdot \frac{d}{dt} \int N dl, \quad (13)$$

where  $e$  and  $m$  are electron charge and mass. Or

$$\Delta f \approx \frac{1}{f} \cdot 0.8 \cdot 10^{-2} \frac{d}{dt} \int N dl \quad (14)$$

#### 4. Estimate of the Magnetic Field in the Corona by the Faraday Effect

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Rotation of the plane of polarization,  $\Psi$ , for plane-polarized radiation is linked with the magnetic field,  $H$ , in the coronas as follows [11]

$$\Psi \approx \frac{1}{4\pi^2 f^2} \cdot 0.93 \cdot 10^6 \int_0^s N H \cos \alpha dl \quad (15)$$

where  $\alpha$  is the angle between the direction and the beam. Reference [11] estimated the maximum angle of rotation,  $\Psi$ , as a function of the impact parameter,  $P$ . At the same time it was assumed that the configuration of the sector structure of the magnetic field was shaped as shown in Figure 8 (that is, that  $\cos \alpha$  has the same sign). And it was taken that  $H$  on the solar surface was  $H_0 = 1$  oersted, that it depends on  $r$  as  $H = H_0 (r/R_\odot)^{-2}$ , and is radial in direction. These assumptions, and the assumption that  $N_0(r)$  has the form of Eq. (3), yield

$$\Psi(P) \approx \frac{10^{24}}{f^2} (0.06 P^{-17} + 0.07 P^{-7} + 2.5 \cdot 10^{-4} P^{-3}), \quad (16)$$

or

$$\Psi(P) \approx (10^3 \cdot 0.08 P^{-7} + 0.3 P^{-3}) \lambda^2 \text{ cm}^2 \quad (17)$$

If the sector structure is turned such that the magnetic field will be along the beam everywhere from the sun (toward the sun),  $\Psi = 0$ . Consequently, when  $P = \text{constant}$ , the plane of polarization will complete an oscillation with a period of several days (weeks) and amplitude of Eq. (17).

Figure 4 shows  $\Psi(P)$  for  $f = 2.29$  GHz. The maximum contribution of the ionosphere to  $\Psi$  for this value of  $f$  is a number of  $7^\circ$ . It is difficult to estimate the contribution of the regular corona to the rotation of the plane of polarization,  $\Psi$ , from the results of the  $\Psi$  measurement for Pioneer 6 eclipsed by the corona because reference [14] does not include the contribution of the ionosphere at the time of the measurements.

#### 5. Fluctuations in the Group Delay and Broadening of the Radio Signal Spectrum in the Turbulent Solar Corona

Fluctuations in the group delay, and broadening of the spectrum of radio waves, can be considered in a geometric optics approximation similar to that made in references [6, 7].

The fluctuations in the index of refraction  $\delta$  (in the electron concentration  $\delta N$ ) [fluctuation in the frequency ( $\Delta n \ll 1$ ) occurs along the path over which the radio waves are propagated] [6] and in  $\delta t_{gr}$

$$\delta t_{gr} = -\frac{1}{c} \int_0^s \delta n(l) dl, \quad (18)$$

$$\delta f = -\frac{1}{\lambda} \int_0^s \frac{d}{dt} \delta n(l) dl, \quad (19)$$

From whence the root-mean-square fluctuations are, respectively

$$\langle \delta^2 t_{gr} \rangle = \frac{1}{c^2} \int_0^s \int_0^s \langle \delta n(l) \delta n(l') \rangle dl' dl \quad (20)$$

$$\langle \delta^2 f \rangle = \frac{1}{\lambda^2} \int_0^s \int_0^s \langle \frac{d}{dt} \delta n(l) \frac{d}{dt} \delta n(l') \rangle dl' dl, \quad (21)$$

Let us direct the  $z$  axis along the direction of beam propagation, and let us say that  $\xi_z = l' - l$ . Let us then say that the correlation length is much shorter than the path  $s$ . Then

$$\langle \delta^2 t_{gr} \rangle = \frac{1}{c^2} \int_0^s \int_{-\infty}^{+\infty} \langle \delta n(l) \delta n(l + \xi_z) \rangle d\xi_z dl, \quad (22) \quad /21$$

$$\langle \delta^2 f \rangle = \frac{1}{\lambda^2} \int_0^s \int_{-\infty}^{+\infty} \langle \frac{d}{dt} \delta n(l) \frac{d}{dt} \delta n(l + \xi_z) \rangle d\xi_z dl \quad (23)$$

But

$$\int_{-\infty}^{+\infty} \langle \delta n(l) \delta n(l + \xi_z) \rangle d\xi_z = \int_{-\infty}^{+\infty} B_n(\xi_z) d\xi_z = 2\pi V(0), \quad (24)$$

where  $B_n(\xi_z)$  is the correlation function,  $V_n$  is the one-dimensional spectrum of the fluctuation in the index of refraction

$$V_n(0) = \int_{-\infty}^{+\infty} V'_n(q) dq = \int_{-\infty}^{+\infty} 2\pi \Phi_n(q) q dq, \quad (25)$$

and  $\Phi_n(q)$  is the three-dimensional spectrum of the fluctuation in the index of refraction described in section 3.

When  $\alpha \gg 1$ , and  $q_2/q_1 \gg 1$ , and using Eqs. (22), (24), and (25), we obtain

$$\langle \delta^2 t_{gr} \rangle = \frac{\pi}{c^2} \frac{3(\alpha-1)}{\alpha} \int_R^{\infty} \langle \delta^2 n \rangle L_1 \frac{r dr}{\sqrt{r^2 - R^2}}, \quad (26)$$

where  $L_1 = 1/q$ . Let us substitute the expression

$$\langle \delta^2 n \rangle \approx \frac{1}{4} \frac{10^{19} \epsilon^2}{16\pi^4 f^4} N^2 = B \cdot N^2, \quad (27)$$

into Eq. (26). Here  $\epsilon^2 N^2 = \langle \delta^2 N \rangle$ . If  $\epsilon = \text{constant}$ ,  $B = \text{constant}$ , whereupon

$$\langle \delta^2 t_{gr} \rangle = \pi \frac{3(\alpha-1)}{c^2 \alpha} B \int_R^{\infty} N^2 L_1 \frac{r dr}{\sqrt{r^2 - R^2}}. \quad (28)$$

Figure 3 shows the results of the numerical estimates of  $(\delta^2 t_{gr})$ .

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Let us assume stability of the Eq. (6) inhomogeneity in order to convert from time-dependent derivatives to spatial derivatives in Eq. (23); that is, we will take it that

$$\left[ \frac{d}{dt} \delta n = -\bar{V} \cdot \nabla \delta n, \right] \quad (29)$$

where  $\bar{V}$  is the wind velocity. We will take it that  $\bar{V}$  is directed along the radius. Then

$$\langle \delta^2 f \rangle = \frac{1}{\lambda^2} \int_0^{\infty} \frac{v^2 R^2}{r^2} \int_{-\infty}^{\infty} \left\langle \left( \frac{\partial}{\partial y} \delta n(l) \right) \left( \frac{\partial}{\partial y} \delta n(l + \xi_z) \right) \right\rangle d\xi_z dl, \quad (30)$$

where OY is the coordinate axis perpendicular to OZ. The integral

$$I = \int_{-\infty}^{\infty} \left\langle \left( \frac{\partial}{\partial y} \delta n(l) \right) \left( \frac{\partial}{\partial y} \delta n(l + \xi_z) \right) \right\rangle d\xi_z$$

can be converted (see [17]) into

$$I = 2\pi^2 \int_0^{\infty} \Phi_n(\alpha, q) q^3 dq. \quad (31)$$

Substituting the model of the spectrum into Eq. (31), and using the smallness  $q_1/q_2 \sim 10^{-4}$ , we can obtain

$$\begin{aligned} I &\approx \frac{\pi}{2} \langle \delta^2 n \rangle \cdot q_2 / \ln(q_2/q_1) \equiv I_1(q_1, q_2) \cdot \langle \delta^2 n \rangle, \quad \text{при } \alpha = 1. \\ I &\approx \frac{3\pi}{2} \left( \frac{\alpha-1}{4 \cdot \alpha^2} \right) \langle \delta^2 n \rangle q_1 \cdot (q_2/q_1)^{2-\alpha} \equiv I_2(\alpha, q_1, q_2) \cdot \langle \delta^2 n \rangle, \quad \text{when } 1 \lesssim \alpha \lesssim 2 \\ I &\approx \frac{3\pi}{8} \langle \delta^2 n \rangle q_1 \ln(q_2/q_1) \equiv I_3(q_1, q_2) \cdot \langle \delta^2 n \rangle, \quad \text{when } \alpha = 2. \end{aligned} \quad (32)$$

Substituting Eq. (32) into Eq. (30)

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$$\langle \delta^2 f \rangle = 2 \frac{1}{\lambda^2} \int_R^{\infty} v^2 N^2 r^4 \frac{R^2}{r^6} \cdot I_i \cdot B \cdot \frac{r dr}{\sqrt{r^2 - R^2}} = 2 \frac{R^2}{\lambda^2} \int_R^{\infty} F^2(r) I_i B \cdot \frac{1}{r^2} \cdot \frac{r dr}{\sqrt{r^2 - R^2}}, \quad (33)$$

where  $i = 1, 2, 3$ .  $F = Nvr^2$  is the flux. We can obtain some estimate of the distribution of the mean plasma flux (of the solar wind) as a function of the heliocentric distance  $r$ , by assuming that  $\langle \delta^2 f \rangle_R$  has been measured and that  $I$  and  $B$  are constants, and by using Eq. (33).

Figure 5 shows the results of the measurements made of the broadening of the Mariner 4 signal spectrum eclipsed by the solar corona [13, 6], and the results of the numerical estimates of  $\sqrt{2\langle \delta^2 f \rangle}$  using Eq. (33). It was taken that  $F = F_0 = \text{constant}$ , and that in the earth's orbit  $vN = 2 \cdot 10^8$  electrons/sec-cm<sup>2</sup>. Further, that  $\epsilon = 0.42$ ,  $\alpha = 2$ ,  $L_1 = 2 \cdot 10^4$  km,  $L_2 = 0.3\rho$  km.

It should be pointed out that Eq. (33), is, as is Eq. (9'), an Abel integral equation with a known inversion.

## 6. Discussion of the Results of the Numerical Estimates.

### Conclusion.

The answer to the question of whether or not it is possible to obtain some mean distribution of solar plasma parameters,  $N_0(r)$ ,  $v(r)$ ,  $H(r)$ , by sounding the corona with monochromatic radio radiation definitely depends on what the corona actually is. If the integral electron concentration is significantly variable, and can change from day to day with a coefficient of  $\sim 2$  (for small impact parameters) the dispersion interferometer method will not provide some mean distribution of electron concentration,  $N_0(r)$ . Measurements of the group delay on two coherent frequencies is more advantageous than the dispersion interferometer method because  $\int N dr$  can be measured directly. These measurements then can be used to obtain some mean distribution of the electron concentration,  $N_0(r)$ . Figures 2 and 3 show the results of the calculated estimates of  $\Delta t_{gr}$  and  $\sqrt{2 \langle \delta^2 t_{gr} \rangle}$  for accepted models of the solar corona. We see that if  $L_1$  and  $L_2$  are proportional to  $r$  ( $L_1 = 2 \cdot 10^4 \rho$  km,  $L_2 = 0.3 \rho$  km),  $\Delta t_{gr}$  and  $\sqrt{2 \langle \delta^2 t_{gr} \rangle}$  will be identically dependent on the impact parameter  $P = R/R_\odot$  as  $P^{-1}$  ( $N_0 \sim r^2$ ) or as  $P^{-1.5}$  ( $N_0 \sim r^{-2.5}$ ). At the same time  $\sqrt{2 \langle \delta^2 t_{gr} \rangle} / \Delta t_{gr} \lesssim 10$  percent. Consequently, such will be the accuracy of the determination of the integral electron concentration of the accepted model of turbulence of the coronal plasma. If  $L_1$  and  $L_2$  do not depend on the heliocentric distance,  $\sqrt{2 \langle \delta^2 t_{gr} \rangle}$  will depend on  $P$  as  $P^{-1.5}$  ( $N_0 \sim r^{-2}$ ) or as  $P^{-2}$  ( $N_0 \sim r^{-2.5}$ ). In this case, when  $P = 4$   $\sqrt{2 \langle \delta^2 t_{gr} \rangle} / \Delta t_{gr} \approx 30$  to 40 percent, and when  $P = 100$  this ratio is  $\sim 10$  percent. As should be anticipated, intensification of turbulence led to an increase in the filtration of the group delay as compared to the results of the Hollweg calculations [5], obtained on the assumption of small-scale fluctuations.

We see from Figure 4 that the contribution of the ionosphere must be taken into consideration when measuring the rotation of the plane of polarization,  $\Psi$ .

The broadening of the Mariner 4 radio signal spectrum in the region  $3 < P \leq 6$  best describes the solar wind turbulence spectrum for  $\alpha = 2$ ,  $L_1 = 2 \cdot 10^4 \rho$  km, and  $L_2 = 0.3 \rho$  km. And it was taken that  $F = N v r^2 = F_0 = \text{constant}$ , in the earth's orbit  $v N = 2 \cdot 10^8$  electrons/cm<sup>2</sup>-sec<sup>-1</sup> and  $\epsilon = 0.42$ .

TABLE 1

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P	$N_0 = 1.5 \cdot 10^8 \rho^{-6} + 2.3 \cdot 10^5 \rho^{-2}$		$N_0 = 1.5 \cdot 10^8 \rho^{-6} + 3 \cdot 10^6 \rho^{-2.5}$	
	$I_1 = \int N d\Omega / \text{cm}^2$	$\frac{\Delta t_{\text{gr}}}{f} \frac{\text{sec}}{100 \text{ MHz}}$	$I_2 = \int N d\Omega / \text{cm}^2$	$\frac{\Delta t_{\text{gr}}}{f} \frac{\text{sec}}{100 \text{ MHz}}$
100	$3.2 \cdot 10^{14}$	$4.2 \cdot 10^{-5}$	$4 \cdot 10^{14}$	$5.2 \cdot 10^{-5}$
10	$4.8 \cdot 10^{15}$	$6.2 \cdot 10^{-4}$	$1.8 \cdot 10^{16}$	$2.3 \cdot 10^{-3}$
5	$1.4 \cdot 10^{16}$	$1.8 \cdot 10^{-3}$	$5 \cdot 10^{16}$	$6.5 \cdot 10^{-3}$
2	$4 \cdot 10^{17}$	$5.2 \cdot 10^{-2}$	$4 \cdot 10^{17}$	$5.2 \cdot 10^{-2}$

TABLE 2

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P	$N_0 = 1.5 \cdot 10^8 \rho^{-6} + 2.3 \cdot 10^5 \rho^{-2}$		$N_0 = 1.5 \cdot 10^8 \rho^{-6} + 3 \cdot 10^6 \rho^{-2.5}$	
	$G_1 \frac{\text{mean}}{\text{cm}^2 \cdot \text{sec}^{-1}}$	$G_2 \text{ cm}^2 \cdot \text{sec}^{-1}$	$G_1$	$G_2$
100-10	$3 \cdot 10^8$	$4 \cdot 10^9 + 6 \cdot 10^{10}$	$1 \cdot 10^9$	$5 \cdot 10^9 + 2 \cdot 10^{11}$
10-5	$1 \cdot 10^{10}$	$6 \cdot 10^{10} + 1.6 \cdot 10^{11}$	$4 \cdot 10^{10}$	$2 \cdot 10^{11} + 6 \cdot 10^{11}$
5-2	$7 \cdot 10^{11}$	$1.6 \cdot 10^{11} + 5 \cdot 10^{12}$	$7 \cdot 10^{11}$	$6 \cdot 10^{11} + 5 \cdot 10^{12}$

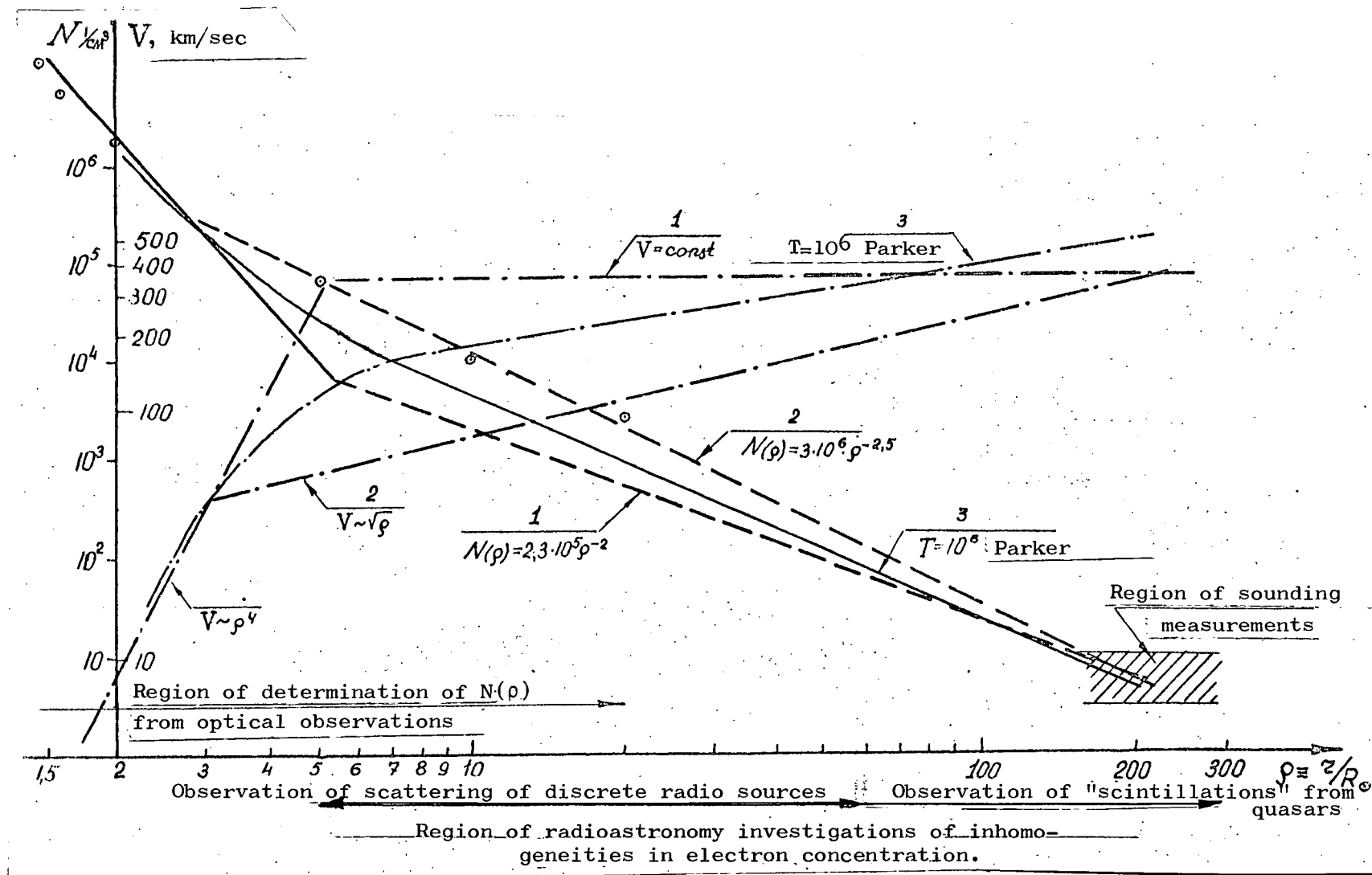


Figure 1. Distribution of electron concentration, and of solar wind velocity in the plane of the ecliptic, in the period of minimum solar activity.

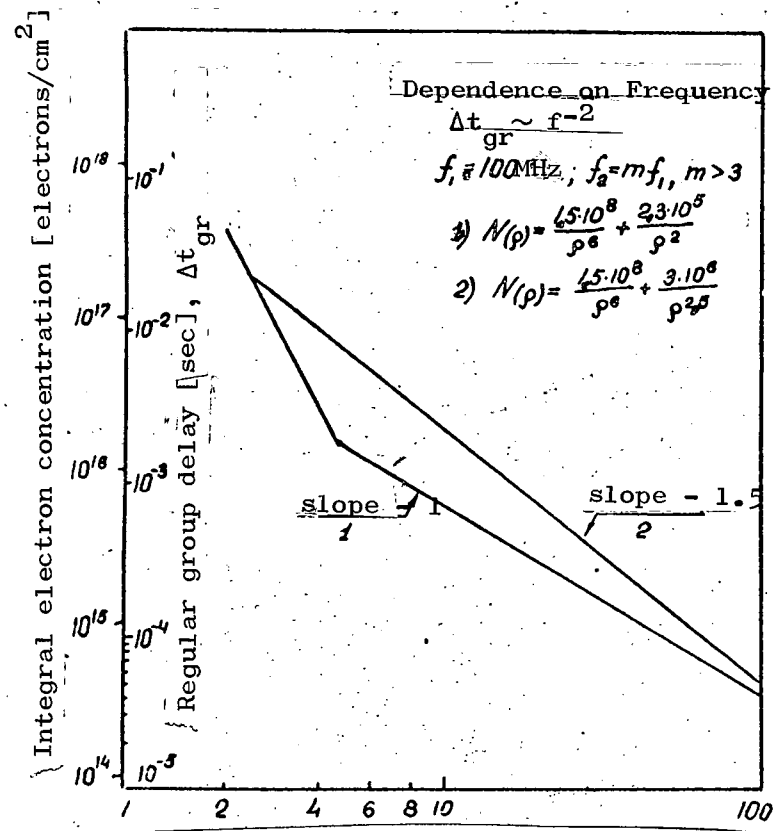


Figure 2. Impact parameter,  
 $R/R_0$

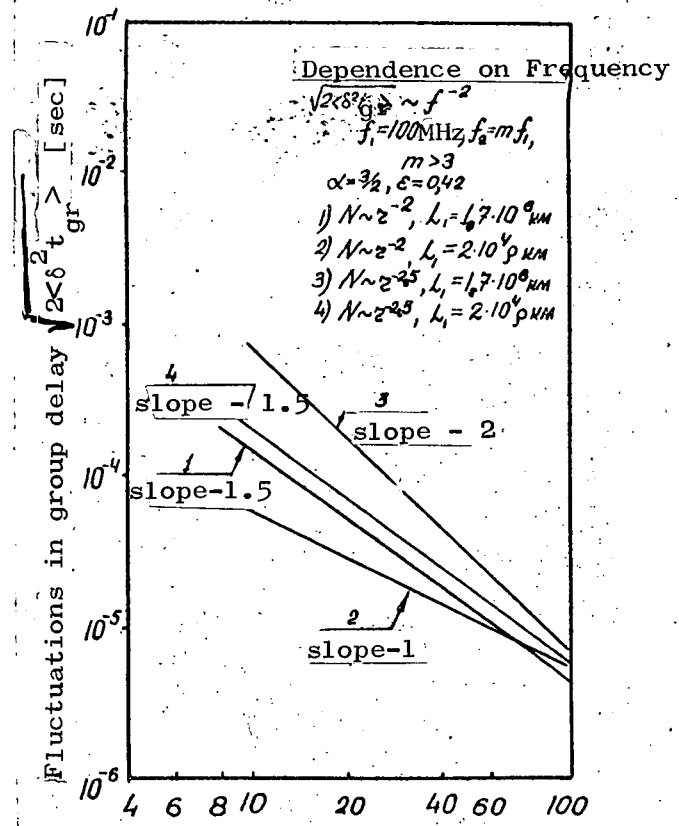


Figure 3. Impact parameter,  
 $R/R_0$

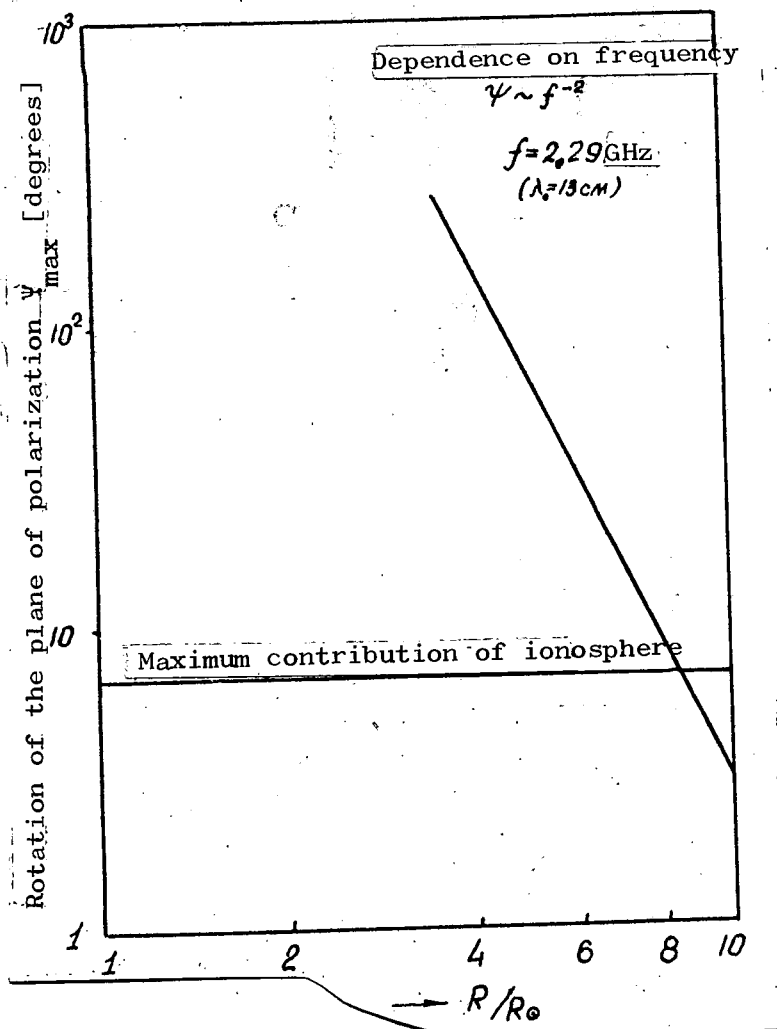


Figure 4.

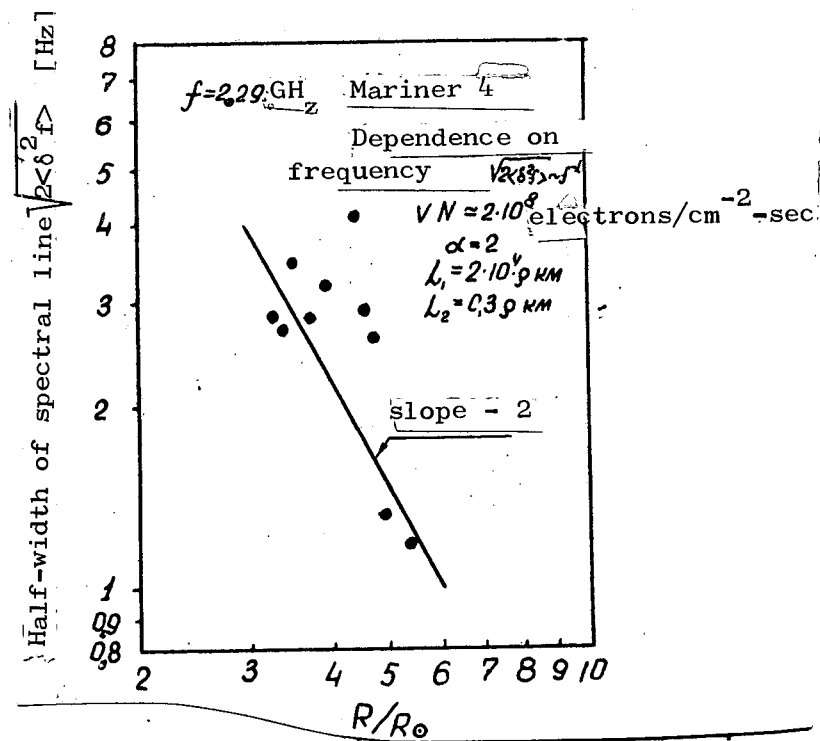


Figure 5.

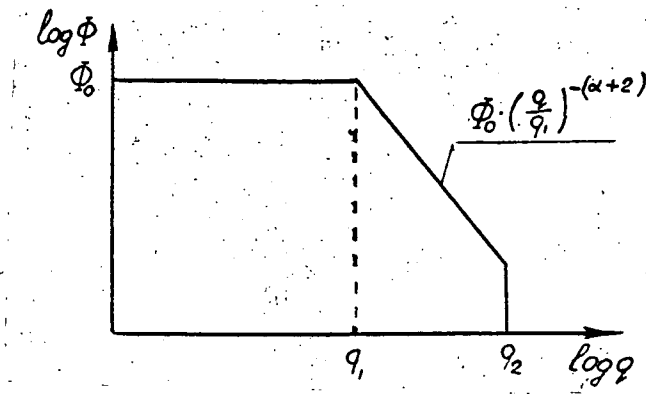


Figure 6.

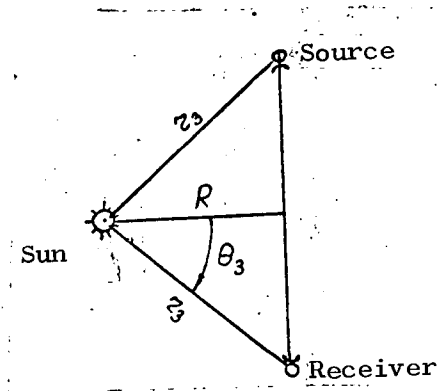


Figure 7.

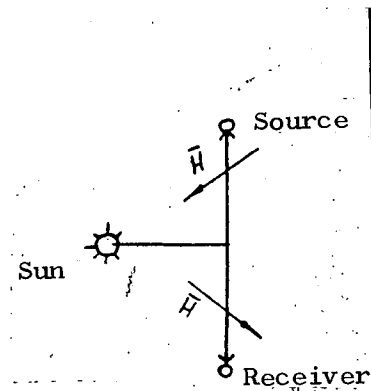


Figure 8.

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